

Solutions

Exam 2 Chapters 5 and 6

Answer the following questions. *You must show your work to receive full credit.* Be sure to make reasonable simplifications. If your answer includes Permutations or Combinations, please find the number it represents. For instance, $C(3, 1) = 3$. Indicate your final answer with a box.

1. (1 point each) Let $A = \{\text{Dirk, Johan, Frans, Sarie}\}$, $B = \{\text{Frans, Sarie, Tina, Klaas, Henrika}\}$ and $C = \{\text{Hans, Frans}\}$.

(a) Find $n(A) + n(C)$. $= 4 + 2 = 6$

(b) Find $n(A \cup C)$. $= n(\{\text{Dirk, Johan, Frans, Sarie, Hans}\}) = 5$

(c) Find $B \cap C$. $= \{\text{Frans}\}$

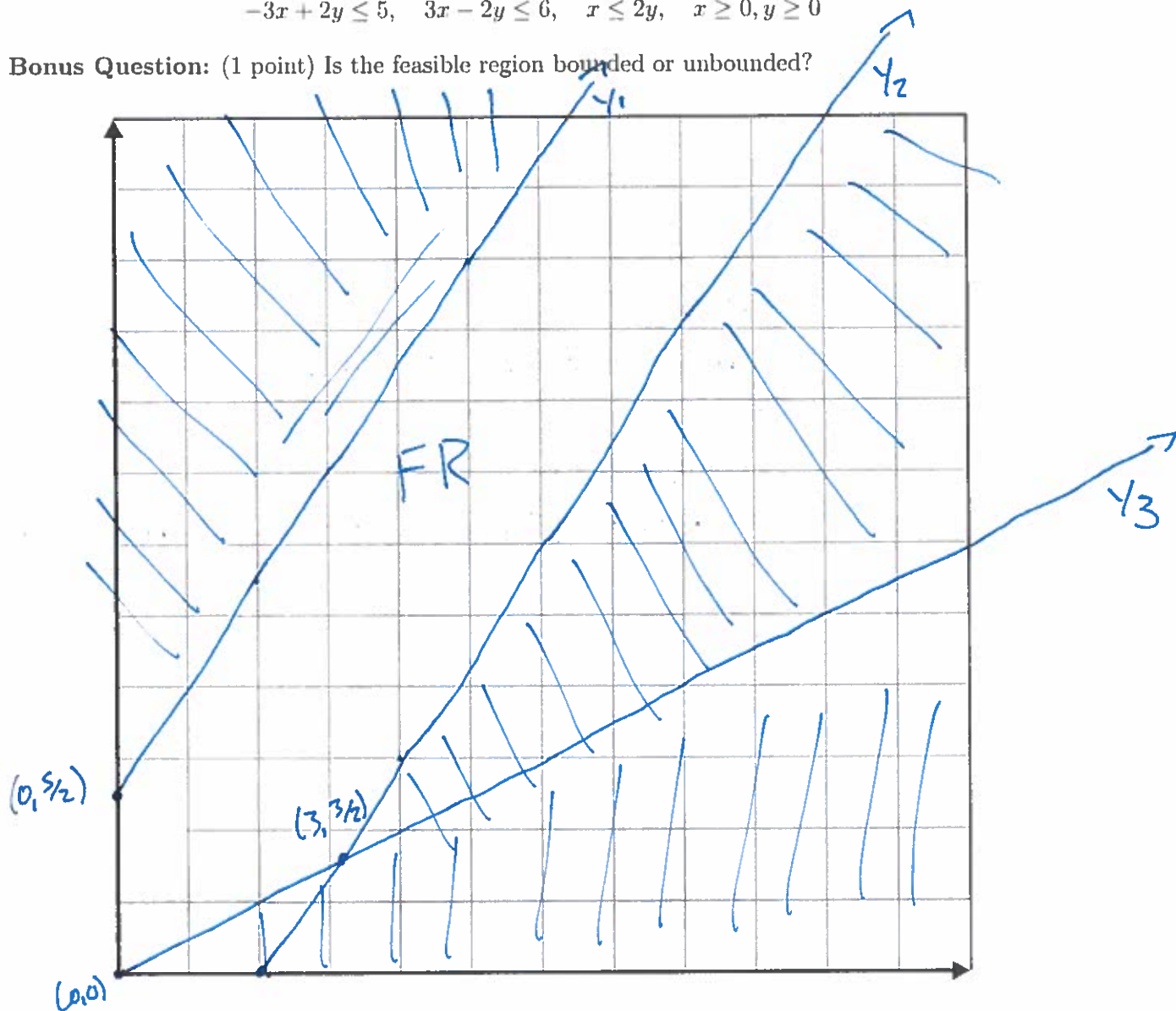
(d) Find $(A \cap B) \cup C$. $= \{\text{Frans, Sarie, Hans}\}$

(e) Find $n(A \cup B \cup C)$. $= n(\{\text{Dirk, Johan, Frans, Sarie, Tina, Klaas, Henrika, Hans}\}) = 8$

2. (10 points) Sketch the following system of inequalities.

$$-3x + 2y \leq 5, \quad 3x - 2y \leq 6, \quad x \leq 2y, \quad x \geq 0, y \geq 0$$

Bonus Question: (1 point) Is the feasible region bounded or unbounded?



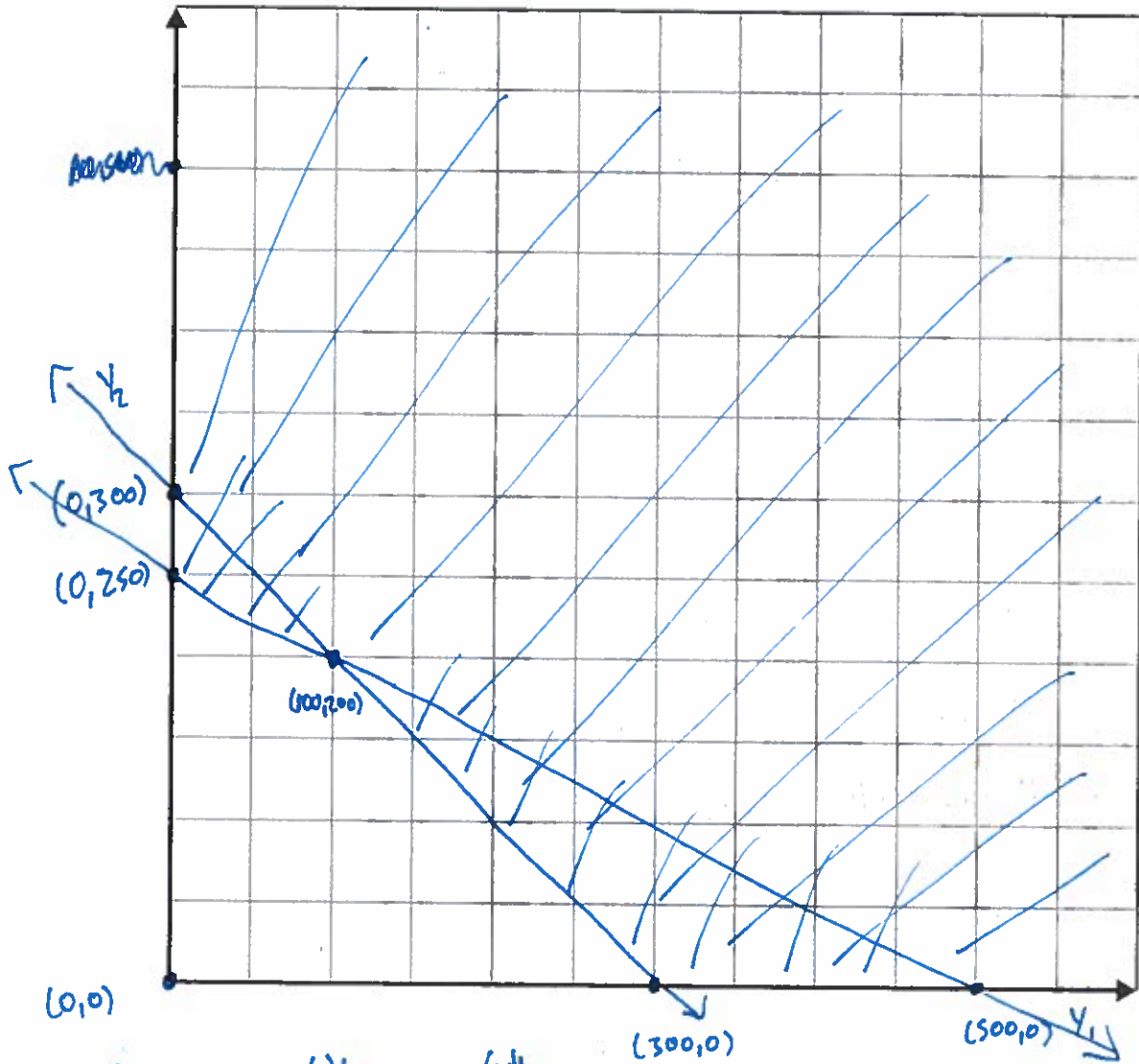
$$-3x + 2y \leq 5 \rightarrow y_1 \leq \frac{5 + 3x}{2}$$

Unbounded.

$$3x - 2y \leq 6 \rightarrow y_2 \geq \frac{3x - 6}{2}$$

$$x \leq 2y \rightarrow y_3 \geq \frac{x}{2}$$

3. (10 points) You manage an ice cream factory that makes two flavors: chocolate and vanilla. Into each quart of vanilla go 2 eggs and 3 cups of cream. Into each quart of chocolate go 1 egg and 3 cups of cream. You have in stock 500 eggs and 900 cups cream. You make a profit of \$3 on each quart of vanilla and \$2 on each quart of chocolate. What is the largest profit and how many quarts of each flavor should you make in order to earn the largest profit?



	Chocolate (x)	Vanilla (y)	Total
Eggs	1	2	500
Cream	3	3	900

$$x \geq 0, y \geq 0$$

$$x + 2y \leq 500 \rightarrow y_1 \leq \frac{500 - x}{2}$$

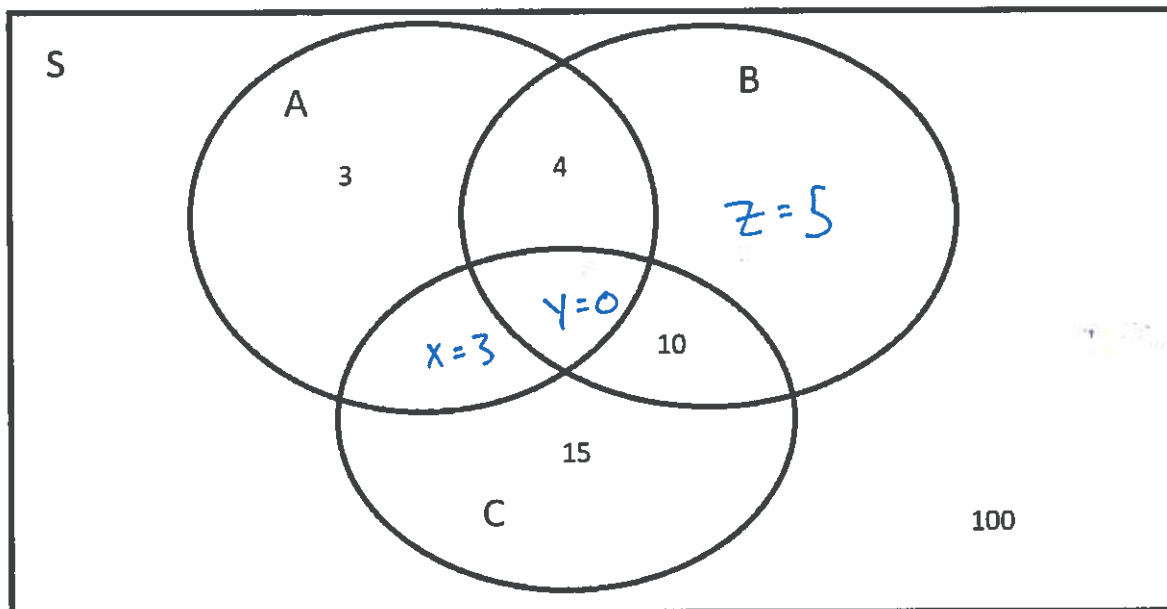
$$3x + 3y \leq 900 \rightarrow y_2 \leq \frac{900 - 3x}{3} = 300 - x$$

$$\text{Profit} = 2x + 3y$$

Corner	Profit
(0,0)	0
(0,250)	750
(100,200)	800
(300,0)	600

Profit is maximized at \$800 when making 100 quarts of chocolate and 200 quarts of vanilla.

4. (6 points) Complete the Venn diagram.



$$n(A) = 10, n(B) = 19, n(S) = 140$$

$$n(A) = 10 = 7 + x + y \rightarrow x + y = 3$$

$$n(B) = 19 = 14 + y + z \rightarrow y + z = 5$$

$$n(S) = 140 = 132 + x + y + z \rightarrow x + y + z = 8$$

Augmented Matrix

$$\begin{pmatrix} 1 & 1 & 0 & 3 \\ 0 & 1 & 1 & 5 \\ 1 & 1 & 1 & 8 \end{pmatrix} \xrightarrow{\text{RREF}} \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 5 \end{pmatrix}$$

So $(x, y, z) = (3, 0, 5)$.

5. (3 points each)

(A) You are searching online for techno music that is neither European nor Dutch. In set notation, which set of music files are you searching for?

$(E \cup D)'$

- (i) $\text{Techno} \cap (\text{European} \cap \text{Dutch})'$
- (ii) $\text{Techno} \cap (\text{European} \cup \text{Dutch})'$
- (iii) $\text{Techno} \cup (\text{European} \cap \text{Dutch})'$
- (iv) $\text{Techno} \cup (\text{European} \cup \text{Dutch})'$

(B) You would like to see either a World War II movie, or one that is based on a comic book character but does not feature aliens. Which set of movies are you interested in seeing?

- (i) $\text{WWII} \cap (\text{Comics} \cap \text{Aliens})'$
- (ii) $\text{WWII} \cap (\text{Comics} \cup \text{Aliens})'$
- (iii) $\text{WWII} \cup (\text{Comics} \cap \text{Aliens})'$
- (iv) $\text{WWII} \cup (\text{Comics} \cup \text{Aliens})'$

Bonus Question: (3 points) Explain, making reference to operations on sets, why the statement "He plays soccer or rugby and cricket" is ambiguous. (If nothing else, come up with two interpretations of the statement.)

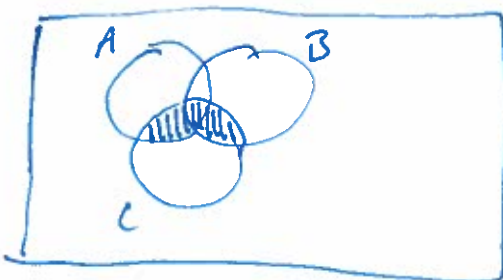
A: He plays soccer

B: He plays Rugby

C: He plays cricket

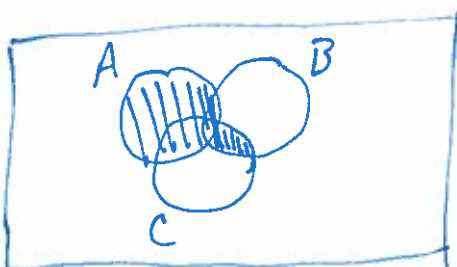
$A \cup B \cap C$

Is it



(i) $(A \cup B) \cap C$ so that he definitely plays cricket and also plays either soccer or rugby or both

Or



(ii) $A \cup (B \cap C)$ so that he either plays soccer or he plays rugby and cricket or possibly all three?

6. Let S be the sample space for the experiment consisting of rolling two (distinguishable) dice and viewing the two face up numbers; i.e. $n(S) = 36$.

- (a) (2 points) Let E be the event that at least one odd die is rolled. Find $n(E)$.
- (b) (2 points) Let F be the event that at least one even die is rolled. Find $n(F)$.
- (c) (4 points) Explain in words what rolls are in the event $E \cup F$ and find $n(E \cup F)$.
- (d) (2 points) Find $n(E \cap F)$.

(a) ~~AAAA~~ $E' =$ no odd die rolled $n(E') = 3 \cdot 3 = 9$

So $n(E) = n(S) - n(E') = 36 - 9 = 27$

(b) Same as part (a). $n(F) = 27$

(c) $E \cup F$ is the event at least one odd die is rolled or at least one even die is rolled.

This consists of all rolls. So $E \cup F = S$ and $n(E \cup F) = 36$

(d) $n(E \cap F) = n(E) + n(F) - n(E \cup F) = 27 + 27 - 36 = 18$.

7. You have a bag containing 3 Knuts, 3 Sickles and 3 Galleons.

- (a) (2 points) Find the total number of combinations possible if you were to draw three coins from your bag.
- (b) (3 points) Count the numbers of ways you can draw one of each different coin when drawing three from the bag.
- (c) (3 points) Count the number of ways, when drawing four coins from the bag, that you can draw exactly two each of two different coins.

$$(a) \ C(9,3) = 84$$

$$(b) \ C(3,1) \cdot C(3,1) \cdot C(3,1) = 3^3 = 27$$

(c) Step 1: Choose types of coins $C(3,2)$

Step 2: Choose 2 of first type $C(3,2)$

Step 3: Choose 2 of second type $C(3,2)$

$3 \cdot 3 \cdot 3 = 27$

8. A credit card number consists of 4 blocks of 4 numbers ranging, and including, 0-9. You are trying to remember yours so you can buy some Nikes.

- (a) (2 points) How many credit card numbers are there?
- (b) (2 points) You remember that the first block of numbers is 1234. How many credit card numbers are left that could be yours?
- (c) (3 points) You now recall that there are exactly two 0's and four 1's. How many credit card numbers are left that could be yours?
- (d) (3 points) Finally, you remember that there is at least one 7 of the remaining numbers. How many credit card numbers are left that could be yours?

(a) $10^{16} = 10,000,000,000,000,000$ 10 quadrillion

(b) $10^{12} = 1,000,000,000,000$ 1 trillion

(c) one 1 is already used in the first block of four.

Step 1: Place the 0's $C(12, 2) = 66 \cdot 1^2$

Step 2: Place the 1's $C(10, 3) = 120 \cdot 1^3$

Step 3: Remaining

~~10~~ 8^7

$66 \cdot 120 \cdot 8^7 = 78,720,000,000$

78 billion 700 million
16,609,443,840
just over 16 1/2 billion

Count the compliment

(d) Step 1: Place the 0's $C(12, 2) = 66 \cdot 1^2$

Step 2: Place the 1's $C(10, 3) = 120 \cdot 1^3$

Step 3: Place the remaining w/o 7's 7^7

$66 \cdot 120 \cdot 7^7 = 6,522,460,560$

$16,609,443,840 - 6,522,460,560 = 10,086,983,280$
just over 10 billion.

You probably won't be getting those nikes.